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The Models of Random Periodic Information Signals on the White Noise Bases

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Abstract—The definition of a random periodic process, the main properties of the process, and a characteristic function for a process of general type that allows us to scope the random periodic process are proposed. Modelling of a linear stochastic process is used as an example for a practical application of a periodic white noise. The characteristic function of nonstationary linear stochastic processes is presented.

Keywords—Random periodic process, Characteristic function, Periodic white noise, Nonstationary linear stochastic process.

We have to deal with nonstationary random processes in applications in many cases. Investigations of the periodic correlated random processes structure and statistical parameters estimations, at the correlation theory level have been developed by Oqura [1]. Random periodic processes were discussed for the first time by Slutskiy [2]. These processes can be defined in the following way. A real random process $\xi(t)$, $t \in (-\infty, \infty)$ is called the real periodic random process, according to Slutskiy, if the fixed number $T > 0$ exists for the process $\xi(t)$, such that the finite dimension vectors $(\xi(t_1), \xi(t_2), \dots, \xi(t_n))$ and $(\xi(t_1 + T), \xi(t_2 + T), \dots, \xi(t_n + T))$ are stochastically equivalent, in a wide sense, for all whole numbers $n > 0$, where $t_1, t_2, \dots$ is a set of separability of the process $\xi(t)$.

The main purpose of this paper is to make the periodic nonstationary random process more exact in an infinitely divisible class by detailed mathematical investigation of the problems connected with the description, definition, and practical application of the process.

According to [3], for the processes with continuous time, the values of the white noise process in a strong sense $\{\zeta(\tau), P[\{\zeta(0) = 0\} = 1]\}$, $\tau \in (-\infty, \infty)$ and those of the process with independent increments $\{\eta(t), t \in (-\infty, \infty)\}$ are connected by the equation

$$\eta(t) = \int_0^t \zeta(\tau)\, d\tau, \qquad t \in (-\infty, \infty); \tag{1}$$

$\zeta(\tau)$ is stochastically equivalent to $\zeta(-\tau)$.

In our case, $\eta(t)$ is a nonhomogeneous Hilbert process with independent increments. There exists a $T > 0$ for the process $\eta(t)$, which is the basis for the following properties' fulfillment:

$$\begin{aligned} &d\kappa_1(t) = d\kappa_1(t + T); \qquad d\kappa_2(t) = d\kappa_2(t + T); \\ &d_x\, d_t L(x, t) = d_x\, d_t L(x, t + T), \qquad \forall t \in (-\infty, \infty), \end{aligned} \tag{2}$$

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX where $\kappa_1(t)$ and $\kappa_2(t)$ are the first cumulant functions of the process $\eta(t), L(x,t)$ is a Poisson jump spectrum in Levy form, T is a correlation period, and $\zeta(\tau)$ is called a periodic white noise. A logarithm of a characteristic function of the real stochastic process $\eta(t)$ in Levy form is

$$\ln f_\eta(u,t) = \left\{ i\,\mu(t)\varepsilon\, u - \frac{D^2(t)}{2}u^2 + \int_{-\infty}^{\infty} \left[e^{iu\varepsilon x} - 1 - \frac{iu\varepsilon x}{1+x^2} \right] d_x L(x,t) \right\}, \qquad t \in (-\infty,\infty), \tag{3}$$

where $\mu(t), D(t)$ are variables, $\varepsilon =$ sign t, and $L(x,t)$ is a Poisson jump spectrum in Levy form.

Nonstationary linear stochastic processes can be defined by application of nonstationary stochastic periodic processes as with independent increments [4]

$$\xi(t) = \int_{-\infty}^{\infty} \varphi(\tau,t)\,d\eta(\tau), \qquad t \in (-\infty,\infty), \tag{4}$$

where $\varphi(\tau,t) \in L_{2,\kappa}$ is a real nonstochastic periodic numerical function on t that has the property

$$\int_{-\infty}^{\infty} \varphi^2(\tau,t)\,d\kappa_2(\tau) < \infty,$$

for every fixed $t \in (-\infty,\infty)$; $\{\eta(t), \eta(0) = 0, t \in (-\infty,\infty)\}$ is called the generating process. The properties (2) are carried out for $\eta(t)$ [3]. Generalization to the complex case proceeds in the traditional way.

Considering the above discussion and according to (2) and (3), the logarithm of the characteristic function of the linear stochastic process (4), in Levy form, is equal [4] to

$$\begin{aligned}\ln f_\xi(u,t) = iu \int_{-\infty}^{\infty} \varphi(\tau,t)\,d\mu(\tau) + \int_{-\infty}^{\infty} \varphi^2(\tau,t)\,dD(\tau) \\ + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[e^{iux\varphi(\tau,t)} - 1 - \frac{i\,u\,x\,\varphi(\tau,t)}{1+x^2} \right] d_x\,d_\tau L(x,\tau),\end{aligned} \tag{5}$$

where

$$\begin{aligned} d\mu(\tau) &= d\kappa_1(\tau) - \int_{-\infty}^{\infty} \frac{x^3}{1+x^2}\,d_x\,d_\tau L(x,\tau), \\ dD(\tau) &= d\kappa_2(\tau) - \int_{-\infty}^{\infty} x^2 d_\tau d_x L(x,\tau), \quad \text{and} \\ d\mu(\tau) &= d\mu(\tau+T), \quad dD(\tau) = dD(\tau+T). \end{aligned}$$

As properties (2) and periodicity of $\varphi(\tau,t)$ are fulfilled,

$$\begin{aligned} \int_{-\infty}^{\infty} \varphi(\tau,t)\,d\mu(\tau) &= \int_{-\infty}^{\infty} \varphi(\tau,t+T)\,d\mu(\tau); \\ \int_{-\infty}^{\infty} \varphi^2(\tau,t)\,dD(\tau) &= \int_{-\infty}^{\infty} \varphi^2(\tau,t+T)\,dD(\tau); \end{aligned}$$

$$\begin{aligned} &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[e^{iux\varphi(\tau,t)} - 1 - \frac{i\,u\,x\,\varphi(\tau,t)}{1+x^2} \right] d_x\,d_\tau L(x,\tau) \\ &\quad = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[e^{iux\varphi(\tau,t+T)} - 1 - \frac{i\,u\,x\,\varphi(\tau,t+T)}{1+x^2} \right] d_x\,d_\tau L(x,\tau+T). \end{aligned}$$

Therefore, the following identity is fulfilled

$$f_\xi(u,t) = f_\xi(u,t+T).$$

The process $\xi(t)$ is a nonstationary random process. Using the characteristic function of a linear stochastic process one can perform a full analysis of the output signals of linear systems, when the input signals are nonstationary random periodic processes. The calculation of the moments and distribution functions, taking into account the properties of the process $\eta(t)$, can be done.

Stochastic periodic processes can be widely used in many applications: the problem of detection of signals from noise, classification of information signals, and many others.

Using the model one can generate pseudostochastic series with the required probabilistic characteristics in infinitely divisible class of distributions.

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